Simple example of total (sys + env) w/ ergodicity + mixing

N+1 spins : ↑ or ↓ ⇒ label spin 1 "system"
remaining N spins "environ."

energy: ϵ or 0
>0

total (sys + env) has energy $E_{tot} = k\epsilon$ (k ↑ spins)

the system (spin 1) has states $E_1 = 0$ (↓)

$E_2 = \epsilon$ (↑)

Dynamics: at every time step δt, choose at random one ↓ spin + one ↑ spin, flip the pair (preserves E_{tot})

⇒ clearly ergodic + mixing, since any configuration of k ↑ spins can evolve in time to any other config. of k ↑ spins

For the system itself:

$W_{12}\,\delta t$ = prob. to go from ↑ to ↓ in time step δt given spin 1 is ↑ $= \frac{1}{k}$ (prob. to choose spin 1 as the ↑ spin to flip)

$W_{21}\,\delta t$ = prob. to go from ↓ to ↑ given spin 1 is ↓ $= \frac{1}{N+1-k}$ (prob. to choose spin 1 as the ↓ spin to flip)

note: $2 \to 1$ (↑ → ↓) leads to loss of energy ϵ from sys. to environment

$1 \to 2$ (↓ → ↑) gain of energy ϵ from environment

Let us check detailed balance:

$$\frac{W_{12}}{W_{21}} \stackrel{?}{=} \frac{\Theta_1}{\Theta_2}$$

Θ_1 = #env. states when sys. is in state 1 $= \binom{N}{k} = \dfrac{N!}{(N-k)!\,k!}$

Θ_2 = " " " state 2 $= \binom{N}{k-1} = \dfrac{N!}{(N-k+1)!\,(k-1)!}$

$$\frac{\Theta_1}{\Theta_2} = \frac{N-k+1}{k} = \frac{W_{12}}{W_{21}}$$ det. balance works!

What is the temperature of the system?

$\Theta_1 = \Theta(E_{tot}) \equiv \binom{N}{E_{tot}/\epsilon}$ since $k = \dfrac{E_{tot}}{\epsilon}$

$\Theta_2 = \Theta(E_{tot} - \epsilon) < \Theta(E_{tot})$ when $E_{tot}/\epsilon \leq \dfrac{N}{2}$