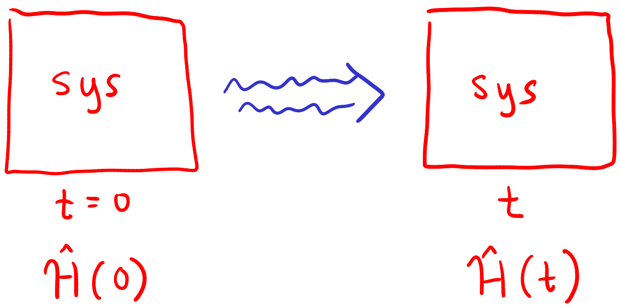
PHYS 414: 4-22-20



one sys: $|\psi(0)\rangle \rightsquigarrow |\psi(t)\rangle = \hat{U}(t)\,|\psi(0)\rangle$

ensemble of sys:

$$\hat{\rho}(0) \rightsquigarrow \hat{\rho}(t) = \hat{U}(t)\,\hat{\rho}(0)\,\hat{U}^{\dagger}(t)$$

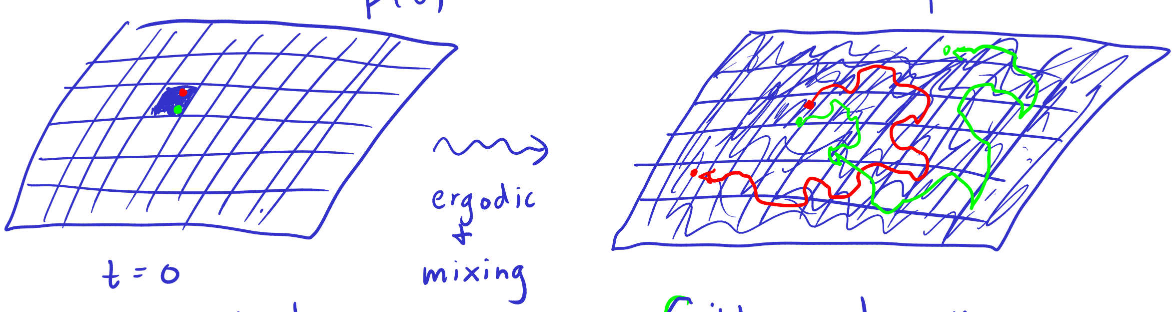
von Neumann entropy:

$$S(\hat{\rho}(0)) = S(\hat{\rho}(t))$$

Why is von Neumann entropy diff. than classical thermo. (Gibbs) entropy in behavior?

classical "pure" ensemble

$\vec{p}(0)$ $\qquad\qquad$ $\vec{p}(t)$



t = 0

100% prob. to be in some phase space microstate

Gibbs entropy

$$S(t) = -k_B \sum_n p_n(t) \ln p_n(t)$$

increases w/ time until reaching max

quantum pure ensemble

all sys in some state $|\psi_1\rangle$

$$\hat{\rho}(0) = |\psi_1\rangle\langle\psi_1| \rightsquigarrow \hat{\rho}(t) = \hat{U}(t)|\psi_1\rangle\langle\psi_1|\hat{U}^\dagger(t)$$
$$= |\psi_1(t)\rangle\langle\psi_1(t)|$$

$|\psi_1\rangle$ contains everything we can possibly know about our quantum sys $\Rightarrow$ complete description (unlike coarse-grained microstate)

B/c von-Neumann is based on quantum states in the ensemble there is no "info" lost during unitary time evolution

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Can von Neumann entropy ever increase?

- for isolated sys obeying unitary time evol $\Rightarrow$ NO
- what about if the sys is being measured?
  $\Rightarrow$ this necessarily means you bring in some outside apparatus to interact w/ sys + hence sys is no longer isolated

example: measuring an observ. corresp. to oper. $\hat{A}$ in an ensemble

fraction: p_1 p_2 ...
ensemble: $|\psi_1\rangle$ $|\psi_2\rangle$

density oper:
$$\hat{\rho} = \sum_n p_n |\psi_n\rangle\langle\psi_n|$$

on measurement

collapse to some e-state $|a\rangle$ w/ prob. $|\langle a|\psi_1\rangle|^2$

$\hat{A}|a\rangle = a|a\rangle$

ensemble after meas. $\quad \overset{p_a}{|a\rangle} \quad \overset{p_b}{|b\rangle} \quad \cdots$

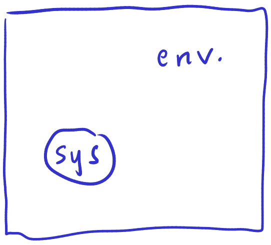
$$P_a = \sum_n p_n \left| \langle a | \psi_n \rangle \right|^2$$

p_n: prob. of $|\psi_n\rangle$ in orig. ensemble

$\left| \langle a | \psi_n \rangle \right|^2$: prob. of $|\psi_n\rangle$ collapsing to $|a\rangle$

new density operator: $\hat{\rho}' = \sum_a P_a |a\rangle\langle a|$

tot = unitary : $\hat{\rho}_{tot}(t)$ evolves unitarily



$S(\hat{\rho}_{tot}(t))$ is const.

$\hat{\rho}_{sys}(t)$ does <u>not</u> evolve unitarily in general