RG Methods in Statistical Field Theory: Quiz 9 Answer

Friday, December 22, 2006

Consider a lattice spinless fermion system described by the following Hamiltonian:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + c_j^\dagger c_i) - \mu \sum_i c_i^\dagger c_i$$

It contains only nearest-neighbor hopping and chemical potential terms. The partition function can be written as a Grassmann path integral:

$$Z = \int e^S \prod_{n=-\infty}^{\infty} \prod_i d\bar{\psi}_i(\omega_n) d\psi_i(\omega_n)$$

where ω_n are the Matsubara frequencies. Write an expression for the action S. (You can just give the answer without any derivation.)

Answer:

$$S = \beta \sum_n \left[\sum_i i\omega_n \bar{\psi}_i(\omega_n)\psi_i(\omega_n) + t \sum_{\langle ij \rangle} \big(\bar{\psi}_i(\omega_n)\psi_j(\omega_n) + \bar{\psi}_j(\omega_n)\psi_i(\omega_n) \big) + \mu \sum_i \bar{\psi}_i(\omega_n)\psi_i(\omega_n) \right]$$

The first term is the Fourier transformed version of $-\bar{\psi}(\tau) \cdot \frac{\partial}{\partial \tau}\psi(\tau)$, while the second and third terms are just $-\mathcal{H}$ with the creation and destruction operators replaced by $\bar{\psi}$ and ψ Grassmann vectors.